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6. AUTHOR(S)
V. E. Zakharov

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Department of Mathematics
617 N. Santa Rita Avenue
University of Arizona
Tucson, Arizona 85721

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In 2001, we (1) (in collaboration with A.Pushkarev) improved and enhanced the Resio-Tracy code for the numerical solution of the Hasselmann's equation; (2) we elaborated (in collaboration with A.Dyachenko and O.Vasyliev) a new version of the exact Euler equation (Dyachenko's equations) for 2-dim potential flow of a fluid with free surface. A new version based on combination of Hamiltonian formalism and conformal mapping, makes possible an efficient use of the Fourier code; (3) we elaborated another code for solution of the MMT model and fulfilled the vast numerical and analytical study of this system. We found that the deviation from the weak-turbulent results can be explained by influence of the coherent structure (quasisolitons and collapses), which is inavoidable in the MMT model. We offered a modification of the MMT model, which demonstrate "restoring" of the weak-turbulent scenario.

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Modeling Swell, High Frequency Spreading and Wave Breaking

V. E. Zakharov

Department of Mathematics, University of Arizona
Tucson, AZ 85721, USA
phone: (520) 6214841 fax: (520) 621-8322 email:zakharov@math.arizona.edu

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LONG-TERM GOALS

My long-term goal is the development of a self-consistent analytical, dynamical and statistical theory of weak and strong nonlinear interactions in ocean gravity waves. The theory will be supported by extensive numerical simulations as well as by laboratory experiments and field observations. The theory will be used as a base for the development of approximate models of Snl, which can be used in a new generation of operational models for wave forecasting. Another goal is the development of the theory of wave breaking, which will make possible to find a well-justified estimate for the rate of energy dissipation due to this process.

OBJECTIVES

The level of nonlinearity in an ensemble of wind-driven ocean waves is relatively small. It makes possible to apply for its statistical description the theory of weak turbulence. In the simplest case, it is the theory of kinetic (Hasselmann's) equation for spectra of a normalized wave action. The kinetic equation has a remarkable family of exact stationary Kolmogorov-type solutions. They are governed by two parameters: fluxes of energy and momentum to the region of high wave numbers, and can be applied for description of energy spectra in the "universal" range behind the spectral peak. All Kolmogorov spectra after averaging in angle have asymptotics $E(\omega) \cong \omega^{-4}$.

The exact kinetic equation is too complicated to be used in the operational models of wave prediction. Thus development of its approximate models is an actual problem.

The wave-breaking, which in most cases participate in the wave dynamics, is a strongly nonlinear effect, making an important contribution to energy dissipation. So far, there is no reliable theory for this phenomenon.

APPROACH

I combine in my work modern analytical methods of mathematical physics with massive numerical simulation and construction of simple phenomenological models. All results are compared with laboratory experiments and field observations.

RESULTS

The last year was extremely productive. Our main achievements are the following:

1. Numerical simulation of the Hasselmann equation.

In 2001 we (in collaboration with A.Pushkarev) improved and enhanced the Resio-Tracy code for the numerical solution of the Hasselmann's equation. A new version of the code makes it possible to use a rather fine mesh (75×36 points) in the frequency-angle plane. The numerical algorithm is stable and fast enough. It allows to follow the evolution of the spectrum of wind-driven waves during as long a period as a dozen hours. Being installed in the update PS, the code performs calculation in one spatial point several times faster than it comes in the real ocean. At the moment we are actively developing a space-time version of the code, which allow the simulation of establishing of spectra in conditions of the limited fetch.

We do not plan to use the new version of the code in operational models. The code will be used soon for checking the calibration of the perspective approximate models of the Snl. In the nearest future we plan to calibrate by the use of the new codes our upgraded second-generation diffusion model. The new code is

used also for verifying the basic concepts of the weak-turbulent (WT) theory. We claim to have achieved a new analytical description of basic phenomena, taking place in the system of interacting waves. The first and foremost prediction of the WT theory is the Kolmogorov spectrum ω^{-4} in the universal range. The equivalent spatial spectrum is $I_k \approx K^{-5/2}$.
Our numerical experiments already strongly support these results.
The results of numerical simulation of the Hasselmann equation are presented in the article [1] to be published in the "Physical Review E".

2. Numerical and analytical study of wave-breaking phenomena and rogue waves.

We elaborated (in collaboration with A.Dyachenko and O.Vasyliev) a new version of the exact Euler equation (Dyachenko's equations) for 2-dim potential flow of a fluid with free surface. A new version based on combination of Hamiltonian formalism and conformal mapping, makes possible an efficient use of the Fourier code.
We started systematic numerical experiments. Even the first results are very impressive. We see, for instance, the formation of freak waves as a result of nonlinear development of the modulational instability of an almost monochromatic Stokes wave. The new method for fast numerical solution of the exact hydrodynamics equations works, in principal, at any level of nonlinearity.
By the use of this method we hope to find soon a well justified expression for dissipation coefficient due to wave breaking.
The results are presented for publication to the "European Journal of Mechanics" [2].

3. Numerical study of one-dimensional weak turbulence.

One of the most important questions in the problem of wave modeling is the applicability of the weak-turbulent Hasselmann equation to the description of real ocean waves. Some authors are sceptical about such a possibility.
It is possible to perform a direct numerical verification of the weak-turbulent approach by comparing it with the direct numerical solution of the Hydrodynamic equation. In full 3-d geometry this program is, so far, unrealistic. However, such comparison is possible in a "toy" one-dimensional model of wave turbulence. This modeling was first performed in 1997 in the Courant Institute by Maida, McLaughlin and Tabak. They offered the so-called MMT model of wave turbulence and performed its massive numerical simulation. Their results are dubious, besides the WT Kolmogorov spectra they observed the spectra of new type (MMT spectra).
We elaborated another code for solution of the MMT model and fulfilled the vast numerical and analytical study of this system. We found that the deviation from the weak-turbulent results can be explained by influence of the coherent structure (quasisolitons and collapses), which is inavoidable in the MMT model.
We offered a modification of the MMT model, which demonstrate "restoring" of the weak-turbulent scenarion.
The results are published [3, 4].

$$U_{tt} \pm U_{xx} = (U^2)_{xx} \pm U_{xxxx}$$

4. Analytic study of the Boussinesq equation.

The fourth-order Boussinesq equation, describing one-dimensional weakly nonlinear waves, is an integrable system. This fact was established by Zakharov in 1974, but was not exploited since this time. In the last year we (in collaboration with L.Bogdanov) undertook a systematic analytical study of the Boussinesq equation. In an extended article (50 pages) we develop the Inverse Scattering technique for the Boussinesq equation, which makes possible to study interaction of solitons, including their merging and collapse.
The article is presented for publication to the "Physica D" [5].

TRANSITIONS

The results can be applied to calculation of the drag coefficient over sea surface and to the problem of remote sensing.

RELATED PROJECTS

US ARMY grant DACA 39-99-C-0018.

REFERENCES

1. K. Hasselmann, On the nonlinear energy transfer in a gravity wave spectrum, J. Fluid Mech., 12 (1982), 481-500.
2. V.Zakharov, V.Lvov and G.Falkovski, Kolmogorov spectra of wave turbulence, Springer-Verlag, 1992.
3. A.Maida, D.McLaughlin, E.Tabak, A one-dimensional model for dispersive wave turbulence, J. Nonlinear Sci. 6 (1997), 9-44.

PUBLICATIONS

[1]. A. Pushkarev, D. Resio and V. Zakharov, Weak turbulence theory, accepted for publication in the "Physical Review E".

[2]. V. Zakharov, A. Dyachenko and O. Vasiliev, New method for numerical simulation of a nonstationary potential flow with a free surface, presented for publication to "European Journal of Mechanics"

[3]. V.Zakharov, P.Guyenne, A.Pushkarev, F.Dias, Wave turbulence in one-dimensional models, Physica D 152-153(2001), 573-619.

[4]. V.Zakharov, O.Vasilyev, A.Dyachenko, Kolmogorov spectra in one-dimensional weak turbulence, Pis'ma v ZhETF, 2(2001), 68-70.

[5]. V. Zakharov and L. Bogdanov, Boussinesq equation revisited, presented to Physica D.

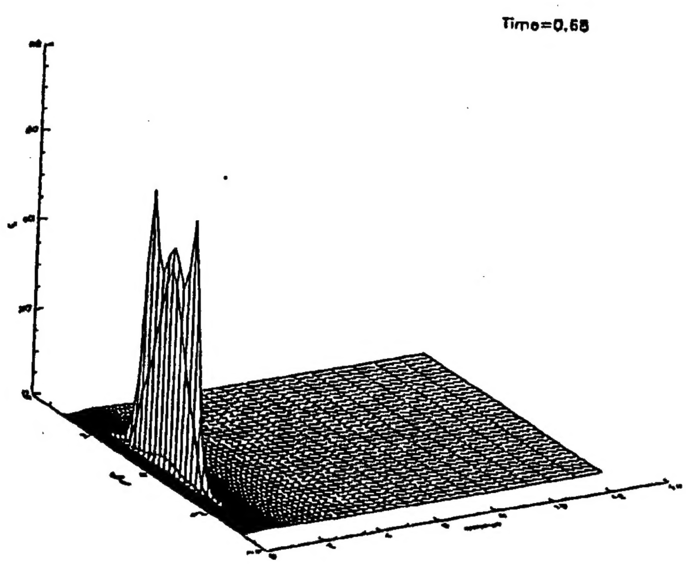

Figure. Wave energy as the function of frequency and angle. (from the Article of Pushkarev, Resio, Zakharov)